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Topic

1. A multi-scale model of fibre reinforced beams using hierarchical one-dimensional finite elements

A multi-scale analysis of fibre reinforced composite beams is proposed by this presentation. At structural level, several higher-order refined beam theories can be easily implemented on the basis of Carrera's unified formulation (CUF) [1] by deriving a fundamental nucleus that does not depend upon the approximation order nor the number of nodes per element (they are free parameters of the formulation).

Under the framework of FE^2 method [2], the effective properties of the fibre-reinforced composite material are found by numerical homogenization over each representative volume element (RVE), that is, the unknown constitutive relationship at the macro-scale is obtained by solving a local finite element problem at the micro-scale. Consequently, a coupled two-scale problem is obtained. Results are validated in terms of accuracy and computational costs towards FEM solutions. Numerical investigations show that accurate results can be obtained with reduced computational costs.

Research Methods: FE^2 , CUF

2. A novel FE^2 method based Fourier macroscopic model for instability phenomena of long fiber reinforced composites

The aim of this work is to develop a computationally efficient multi-scale model to accurately simulate and analyze the instability phenomena of long fiber-reinforced composites.

Towards this end, the Fourier-related double scale analysis and the multi-level finite element method are combined to reduce the computational cost. Although the FE^2 method permits to save a lot of degrees of freedoms, its computational cost is still expensive due to the fact that all the RVEs on the Gauss points of macroscopic model need a sufficient fine mesh to capture the local buckling of long fibers. To overcome this difficulty, the Fourier-related double scale analysis is carried out on the RVEs, where all the microscopic unknowns are developed by Fourier series and the new RVE model is based on the envelopes of the microscopic unknowns. This Fourier development has two advantages: 1) only the envelopes of instability patterns are evaluated and this leads to a significant improvement on computational efficiency, especially when dealing with high wavenumber buckling phenomena; 2) the new RVE model allows one to select modal wavelength, which makes it easy to control non-linear calculations. The established non-linear multi-scale system is solved by the Asymptotic Numerical Method (ANM), which is more reliable and less time consuming than other iterative classical methods.

Research Methods: FE^2 , ANM, Techniques of Fourier series with slowly varying coefficients

Motivation

1. Fiber-reinforced materials are widely used in different engineering sectors.
2. The macroscopic properties of them mainly depend on microstructures and components.
3. The understanding and modelling of instability phenomena is very important for design.

Methods

Carrera Unified Formulation:

1. The displacement field is a priori assumed in the following manner:

$$\mathbf{u}(x, z) = F_\tau(z) N_i(x) \mathbf{q}_{\tau i} \quad \text{with } \tau = 1, 2, \dots, N_u, i = 1, 2, \dots, N_n^e$$

According to Einstein's notation, subscript τ implicitly represents a summation. $F_\tau(z)$ is a generic expansion function over the direction orthogonal to beam axis and N_u is the number of accounted terms. $N_i(x)$ is a C^0 shape function, N_n^e the number of nodes per element and $\mathbf{q}_{\tau i}$ the nodal displacement unknown vector. Linear, quadratic and cubic elements based on Lagrangian shape functions are considered. They are referred to as "B2", "B3" and "B4", respectively.

FE2 Method:

For addressing the multi-scale problems, this method can capture the microscopic information compared to the classical homogenization technique, and also save lots of DOFs compared to the full microscopic model.

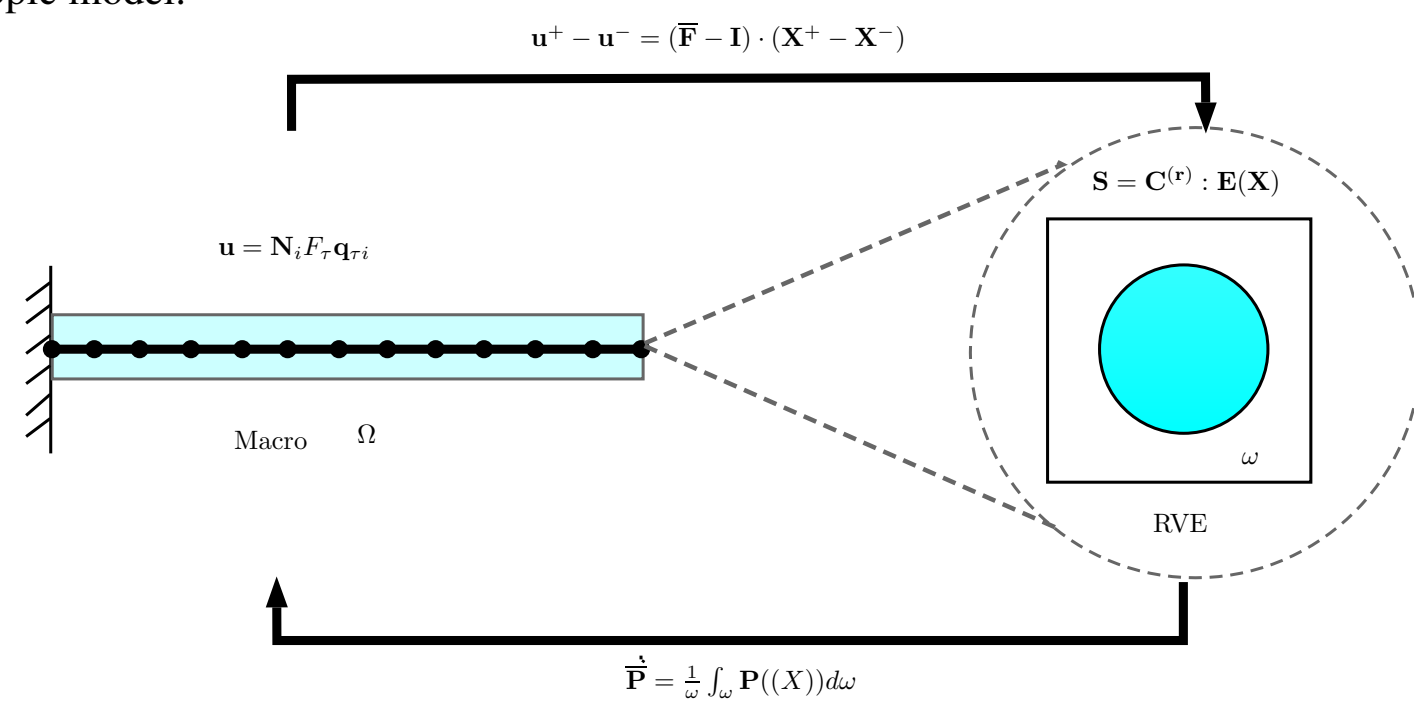
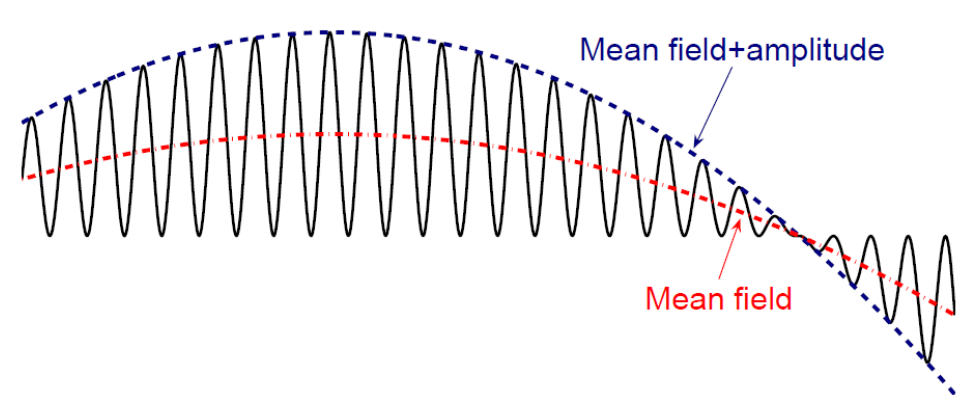


Figure 1: Exchange Information Between Two Scales

Techniques of Fourier series with slowly varying coefficients

Transform the fast oscillation unknown fields to slowly variable fields.

$$U(x) = \sum_{j=-\infty}^{+\infty} U_j(x) e^{jqx} \quad \text{with } j = 0, 1, \dots$$

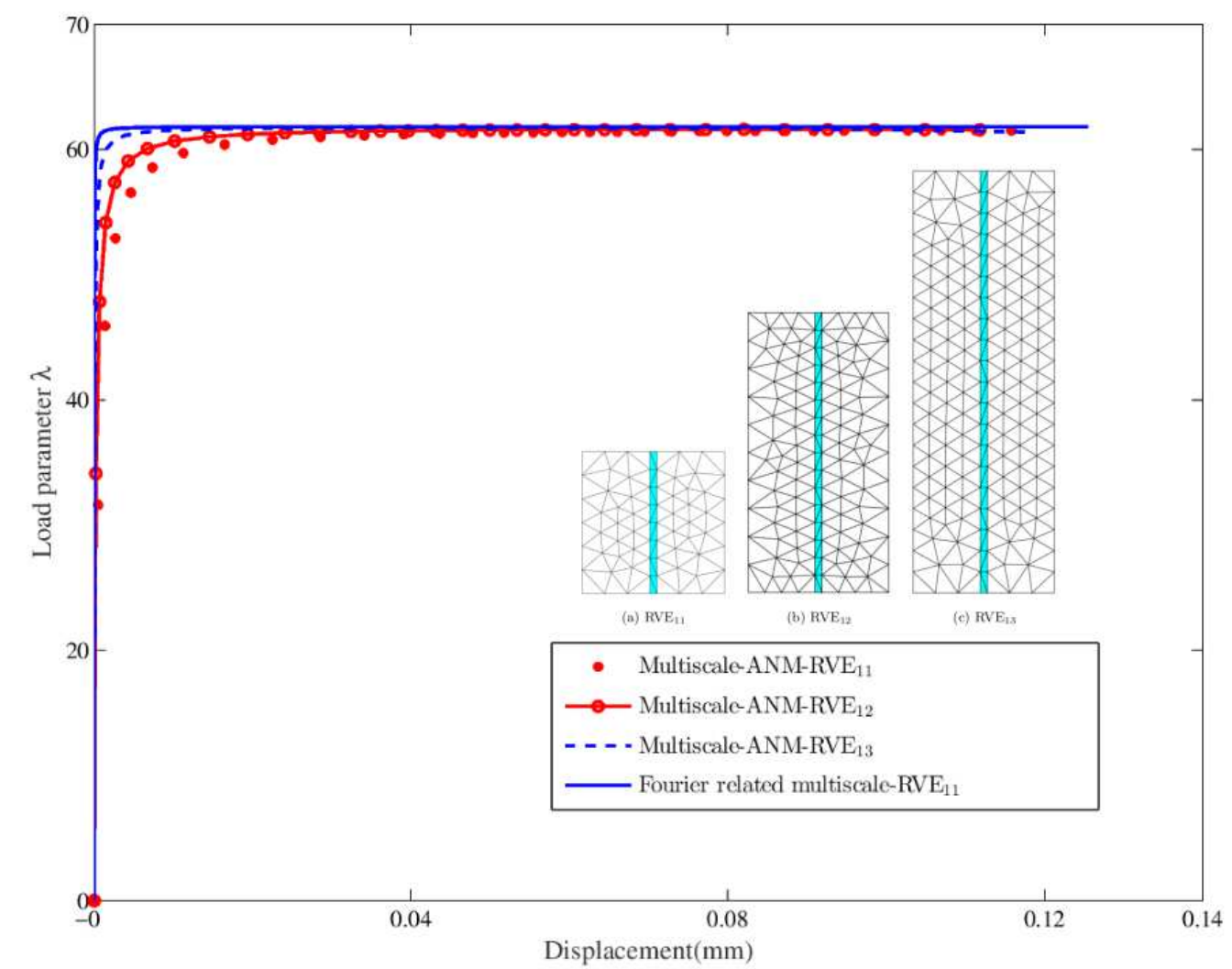


Asymptotic Numerical Method

It only needs to inverse one stiffness matrix in one non-linear step, and it is also very efficient in the case of strong non-linear calculations due to the property of step length self-adaptive.

Results

The load-displacement diagrams by using different RVEs, the wavenumber $q=2$.



Displacement u_x variation of RVE from the cantilevered short beam, at a gausspoint around (0, b/2)

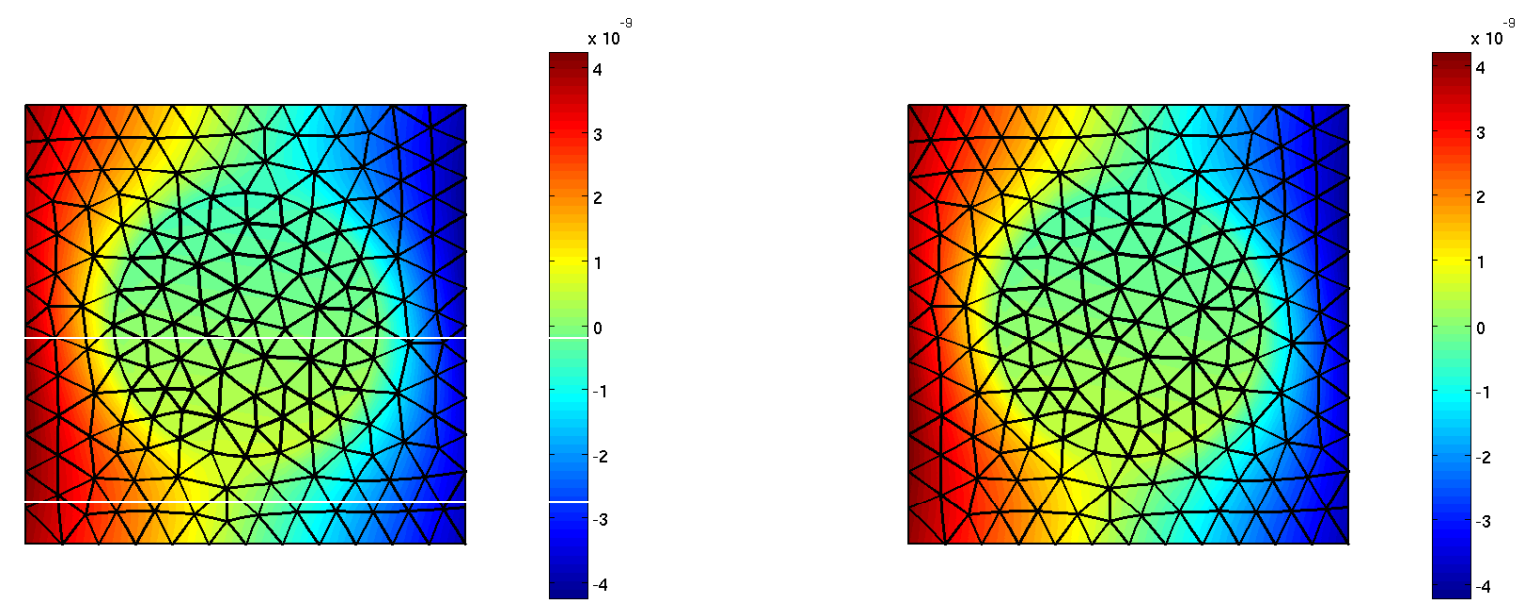


Figure 2: FE^2

Figure 3: Multiscale-CUF

Cases	Simply supported beams	Cantilever beams	Clamped-clamped beams
S=100	1.68% ($N \geq 8$)	1.68% ($N \geq 8$)	1.69% ($N \geq 8$)
S=10	1.55% ($N \geq 8$)	0.29% ($N \geq 8$)	0.31% ($N \geq 8$)

Table 1: Relative errors of displacements between FE^2 model and multiscale-CUF model.

Conclusions

- By introducing the Fourier series with slowly varying coefficients into the representative volume element, a new multiscale model is developed in the frame of the FE^2 method and ANM method.
- By comparing the results with two-dimensional finite elements solution obtained through the Multiscale-FEM and commercial code Abaqus, it can be concluded that higher-order theory match well with the reference solutions, both in macroscopic and Microscopic scale.

Forthcoming Research

Based on the theory of ANM, the model based on the framework of FE^2 and CUF will be modified to be applied for nonlinear cases.

References

- [1] Erasmo Carrera, Gaetano Giunta, and Marco Petrolo. *Beam structures: classical and advanced theories*. John Wiley & Sons, 2011.
- [2] Frédéric Feyel and Jean-Louis Chaboche. Fe 2 multiscale approach for modelling the elastoviscoplastic behaviour of long fibre sic/ti composite materials. *Computer methods in applied mechanics and engineering*, 183(3):309–330, 2000.

Acknowledgements

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